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Simulation of Interaction Between a Flexible Filament and Fluid Flow by Immersed Boundary Method

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Abstract

Simulations of interaction between a flexible filament and fluid flow were undertaken using an immersed boundary method. A hanging filament immersed in fluid flow was simulated for validation. The movement of the flexible filament in oscillating flow was also predicted numerically. Shedding vortices due to the flapping filament were described in this study.

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Keywords: Immersed boundary method; solid-fluid interaction; flexible filament; Dirac delta function; self-sustained flapping.

1. Introduction

A flexible filament flaps in fluid flow is a typical fluid-structure interaction problem. The Immersed Boundary (IB) method has been applied to a wide range of fluid-structure interaction problems (e.g. Zhu and Peskin [1], Huang *et al.* [2]). The incompressible Navier-Stokes equations, which are used to describe the fluid motion, is computed based on Eulerian variables and is defined on a fixed Cartesian mesh in the IB method. This study aims to use the IB method to predict the flapping motion of the filament. The filament motion is described by a Lagrangian frame. The Eulerian and Lagrangian variables are connected by a smoothed approximation of the Dirac delta function as proposed by Peskin [3]. No only uniform flow but oscillating flow is considered to observe the interaction with the filament.

2. Mathematical formulae and numerical methods

2.1. Governing equations for the fluid flow

The incompressible viscous fluid flow is governed by the Navier-Stokes equations and the continuity equation.

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{uu}) = -\nabla p + \frac{1}{Re}\nabla^2 \mathbf{u} + \mathbf{f}. \tag{1}$$

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and

$$\nabla \cdot \mathbf{u} = 0. \tag{2}$$

In Eq. (1), the Navier-Stokes equations is incorporated with the forcing term , **f**, in the dimensionless form. The terms **u** and p are non-dimensional velocity and pressure, respectively. The forcing term **f** is determined by

$$\mathbf{f}(\mathbf{x}, t) = \rho_C \int_\Gamma \mathbf{F}(\mathbf{s}, t)\delta(\mathbf{x} - \mathbf{X}(s, t))\mathrm{d}\mathbf{s}. \tag{3}$$

where

$$\rho_C = \frac{\rho_s - \rho_f \cdot A}{\rho_f L_r} = \frac{\rho_{sf}}{\rho_f L_r}. \tag{4}$$

It should noted that **F** is the Lagrangian force which calculate by the feedback law, A denotes the sectional area of filament and ρ_C is dimensionless density constant representing the ratio of the density difference between the filament and the surrounding fluid.

2.2. Dimensionless equation of motion of flapping flexible filament

Given that the filament is fixed at one end and the other is free, the dimensionless equation of motion of the flapping filament is described as:

$$\frac{\partial^2 \mathbf{X}}{\partial t^2} = \frac{\partial}{\partial s}\left(T\frac{\partial \mathbf{X}}{\partial s}\right) - \frac{\partial^2}{\partial s^2}\left(\gamma\frac{\partial^2 \mathbf{X}}{\partial s^2}\right) + Fr\frac{\mathbf{g}}{g} - \mathbf{F} \ , \tag{5}$$

where

$$\mathbf{F} = -\frac{\partial^2}{\partial s^2}\left(\gamma\frac{\partial^2 \mathbf{X}}{\partial s^2}\right) \tag{6}$$

and Fr is the dimensionless Froude number which represents gravity as oppose to inertia and $g = |\mathbf{g}|$. At the free end, the boundary conditions are:

$$T = 0, \frac{\partial^2 \mathbf{X}}{\partial s^2} = (0, 0) \text{ and } \frac{\partial^3 \mathbf{X}}{\partial s^3} = (0, 0). \tag{7}$$

At the fixed end, the boundary conditions are:

$$\mathbf{X} = \mathbf{X}_0 \text{ and } \frac{\partial^2 \mathbf{X}}{\partial s^2} = (0, 0). \tag{8}$$

The dimensionless interaction force between the fluid and the immersed filament can be calculated by the feedback law, which is described as:

$$\mathbf{F} = \alpha \int_0^t (\mathbf{U}_{ib} - \mathbf{U})\,\mathrm{d}t + \beta\,(\mathbf{U}_{ib} - \mathbf{U})\,, \tag{9}$$

where α and β are constants. The terms α and β are $-\kappa$ and $-\kappa\Delta t$, respectively. The term κ is a positive stiffness constant $\gg 1$. $\mathbf{U}_{ib}$ is the dimensionless fluid velocity obtained by interpolation at the IB and **U** is the dimensionless velocity of the filament expressed by $\mathbf{U} = \mathrm{d}\mathbf{X}/\mathrm{d}t$. The transformation between the Eulerian and Lagrangian variables can be realized by the Dirac delta function. The interpolation of dimensionless velocity is expressed as:

$$\mathbf{U}_{ib}(s, t) = \int_\Omega \mathbf{u}(\mathbf{x}, t)\delta(\mathbf{x} - \mathbf{X}(s, t))\mathrm{d}\mathbf{x} \ \ . \tag{10}$$

The elastic boundary is represented by a set of Lagrangian points. The singular force at the Lagrangian points is determined by the Hooke's law. Force is spread to the surrounding Eulerian points using a delta function. In this study, a 4-point function is used. Calculation of $\delta_h(r)$ is used to solve Eqs. (10) and (3), where r is the parameter representing the position of the submerged Lagrangian points and is scaled with respect to the grid size h; the delta function is described as mentioned by Lee and Kim [4]:

$$\delta_h(r) = \frac{1}{h^2}\phi\left(\frac{x}{h}\right)\phi\left(\frac{y}{h}\right) \ \ . \tag{11}$$

3. RESULTS AND DISCUSSION

3.1. A hanging filament without ambient fluid under a gravitational force

The flapping motion was simulated using Eq. (5) with the boundary conditions given by Eqs. (7) and (8). The hydrodynamic force was neglected. The filament was initially inclined. At $t = 0$, the filament starts to move downward and swings due to the gravitational force. In this simulation, $L = 1$, $KK = 100$, $Fr = 10.0$, and initial perturbation was 0.1π. For $\gamma = 0$, the free end of the filament rolls up at the end of the downward stroke. This phenomenon is known as a 'kick' as mentioned by Huang *et al.* [2]. The 'kick' behaviour was observed the whole time. For $\gamma = 0.01$, the kick behaviour is not observed. This is in good agreement with the result published by Huang *et al.* [2].

3.2. A flexible filament flapping along a uniform flow direction

Subsequently, the flexible filament was immersed in fluid flow. The computational domain used is $0 \leq x \leq 8$ and $0 \leq y \leq 8$. The length of the filament is 1. 513×513 grids in the streamwise and transverse directions were used. The parameters $\alpha = -10^5$; $\beta = -10^2$, and $\Delta t = 0.0003$ were adopted. Fig. 1 shows the instantaneous vorticity contours at a number of selected time steps. For the initial perturbation of 0.1π, the vortex shedding from the filament is split into two vortices by the bending of the free end of the filament.

Simulations of the filament with different bending rigidities show that the filament tends to bend to a greater degree at the free end when γ is low. Vorticity contours for the filament at $Re = 500$ and $\gamma = 0.0001$ shows that each vortex is split into two smaller vortices by the bending of the free end, and the two positive and two negative vortices are shed sequentially from the filament. The production of a small vortex which sheds from the free end of the filament is a combined effect of the Reynolds number and bending rigidity. This trend is consistent with Huang *et al.* [2].

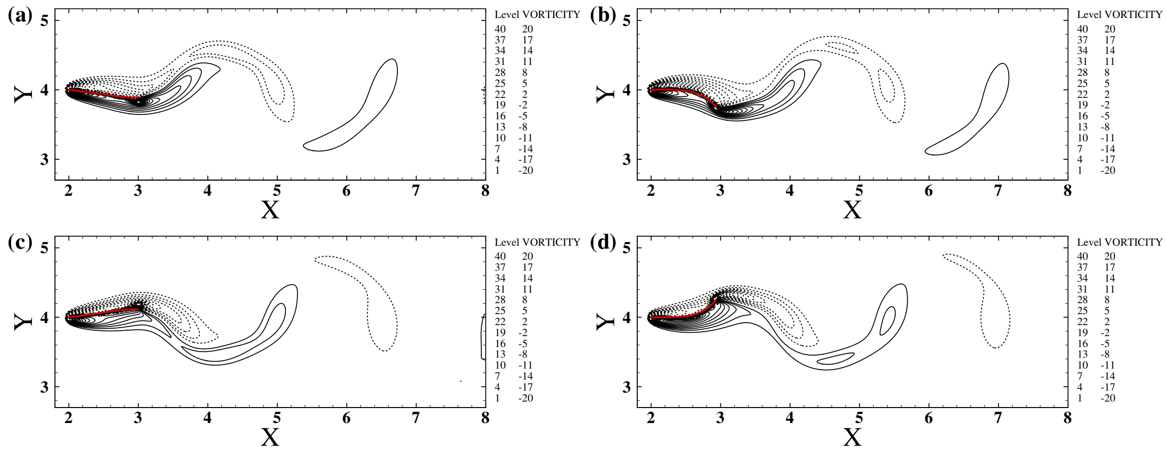

Fig. 1. The instantaneous vorticity contours of the filament in a uniform flow at $Fr = 0.5$, $KK = 64$, $Re = 200$, $\gamma = 0.001$, and $\rho = 1.5$. a) $t = 11.64$; b) $t = 12.12$; c) $t = 13.32$; d) $t = 13.8$.

In all of the simulations of the filament, a self-sustained flapping state quickly develops with an initial perturbation $= 0.1\pi$. For all the simulations with $L = 1$, the flapping state eventually develop no matter how small the initial perturbation. The comparison of the free end position of the filament as reported by Huang *et al.* [2] and the present study at different Re and ρ shows slight discrepancy between the two studies.

3.3. A flexible filament flapping across oscillating flow direction

Furthermore, the filament was immersed in oscillating flow as shown in Fig. 2. The computational domain was $0 \leq x \leq 8$ and $0 \leq y \leq 8$. The length of the filament was 1. 513×513 grids in the streamwise and transverse directions were used. $Re = 100$, $\alpha = -10^5$, $\beta = -10^2$, $\rho = 1$, $KK = 64$, $\gamma = 0.001$, and $\Delta t = 0.0002$ were considered in this study. The horizontal oscillating velocity is $u = 1.5\frac{y}{H_s}(2 - \frac{y}{H_s})\sin\left(\frac{2\pi}{T_C}t\right)$, where T_C is the dimensionless period for the flow and is equal to 10 . Fig. 2 shows the instantaneous velocity vectors in half a cycle. It is found that the filament deflects to the right as the flow rate increases and bounce back as the flow decreases. The reverse flow drives the filament to deflect to the left. This trend occurs periodically.

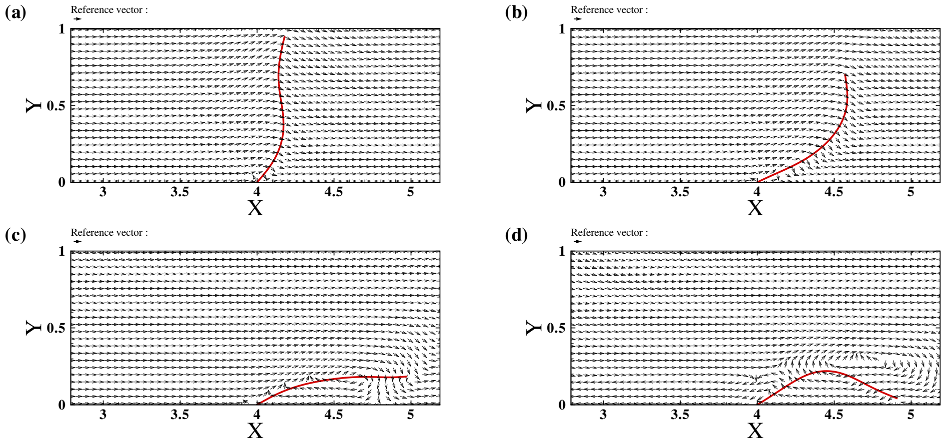

Fig. 2. The instantaneous velocity vectors of the filament in oscillating flow at $KK = 64$, $Fr = 0.0$, $Re = 100$, $\gamma = 0.001$, and $\rho = 1$. a) $t = 1.0$; b) $t = 2.0$; c) $t = 3.0$; d) $t = 4.0$.

4. Conclusions

The flapping motion of a flexible filament in fluid flow has been simulated by the proposed immersed boundary method successfully. The bending rigidity influences the flapping amplitude of the filament. Smaller bending rigidity results to a larger flapping amplitude. Two positive and two negative vortices are shed sequentially from the filament due to increasing Reynolds number. A small vortex is shed from the free end of the filament resulting from a combined effect of the Reynolds number and bending rigidity. A self-sustained flapping state quickly develops for all simulations. The stable stretched-straight state of filament disappears and the flapping state of filament remains if L is sufficiently large. A self-sustained flapping state also occurs due to the influence of the oscillation of the flow along the filament. Compared to the literature, the results from this present method are in good agreement.

Acknowledgements

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